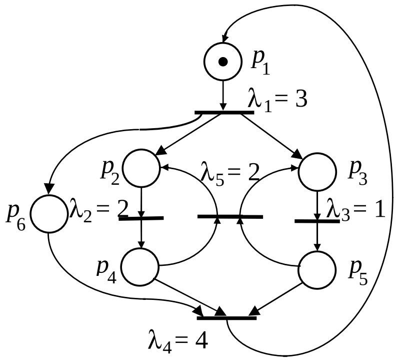
RTDS408 Tutorial Problem and Solutions #4 - Time Augmented and Stochastic Petri nets

1. We have a data acquisition system that has a single data channel acquired at a clock rate of T_1 = 4 time units. The acquisition time is T_2 = 3 time units and the data is pre-processed by different processes that are activated on alternate data points. These different processes perform data conversion (T_3 = 4 and T_5 = 3 time units respectively) and data normalization (T_4 = 5 and T_6 = 6 time units respectively). The data is then combined (T_7 = 1 time unit) and then a moving average filter process is performed (T_8 = 2 time units). Old data used in the filter process is removed by another process (T_9 = 3 time units) and the final stage records the processed data point (T_{10} = 4 time units).

   For this system, produce a Petri net graph to model all processes and augment it with process time information. Derive the constraints imposed on the process times making use of the notion of *safeness in the presence of time*. Determine if the system can achieve the specified time constraints.

2. Given the following SPN model for a system with the specified transition rates, determine the average time for token return to place p_1.



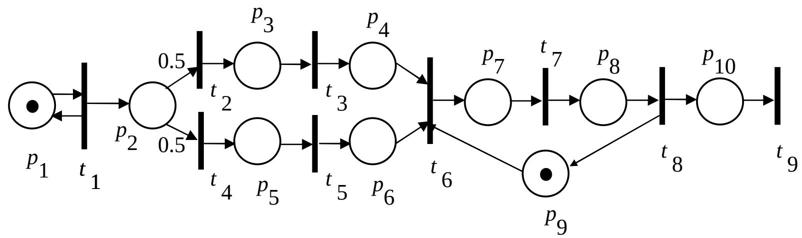
3. Given the control software for a two machine flexible manufacturing system with one assembly process each sharing a common tool, and the following transition rates:

   λ_1 = 6 (process 1 waiting)
   λ_2 = 7 (process 2 waiting)
   λ_3 = 9 (process 1 active)
   λ_4 = 10 (process 2 active)

   On average, what fraction of time is the first process using the shared tool as a percentage of the total assembly time?

Solutions:

1.



Apply consistency balance to all transitions to find all transition MRFFs F_i.:

Let $F_1 = 1 \rightarrow F_2 = 0.5$, $F_4 = 0.5$
$\rightarrow F_3 = 0.5$, $F_5 = 0.5 \rightarrow F_6 = 0.5$
$\rightarrow F_7 = 0.5 \rightarrow F_8 = 0.5 \rightarrow F_9 = 0.5$

Then apply the safeness criteria to each net construction:

a. p_4 and p_6 are entry places to an independent cycle of p_7, p_8 and p_9:
$T_4 \le T_1/F_3$ and $T_7 + T_8 + T_9 \le T_1/F_3 \Rightarrow T_4 \le 2T_1$ and $T_7 + T_8 + T_9 \le 2T_1$
$T_6 \le T_1/F_5$ and $T_7 + T_8 + T_9 \le T_1/F_5 \Rightarrow T_6 \le 2T_1$ and $T_7 + T_8 + T_9 \le 2T_1$

b. p_4 and p_6 are also final places in a parallel synchronous path p_2, p_3, p_4, and p_2, p_5, p_6:
$T_2 + T_3 + T_4 - (T_2 + T_5) \le T_1/F_5 \Rightarrow T_3 + T_4 - T_5 \le 2T_1$
$T_2 + T_5 + T_6 - (T_2 + T_3) \le T_1/F_3 \Rightarrow T_5 + T_6 - T_3 \le 2T_1$

c. p_2, p_3, p_5, p_7, p_8, and p_{10} are all simple places so that:
$T_2 \le T_1/F_1$, $T_3 \le T_1/F_2$, $T_5 \le T_1/F_4$, $T_7 \le T_1/F_6$, $T_8 \le T_1/F_7$, $T_{10} \le T_1/F_8$

Combining all time constraints and inserting the numerical values values:
$T_1 = 4$, $T_2 = 3$, $T_3 = 4$, $T_4 = 5$, $T_5 = 3$, $T_6 = 6$, $T_7 = 1$, $T_8 = 2$, $T_9 = 3$, $T_{10} = 4$

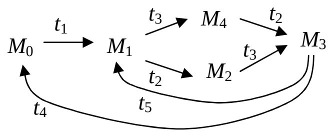
$T_4 \le 2T_1 \Rightarrow 5 \le 2(4)$ and $T_6 \le 2T_1 \Rightarrow 6 \le 2(4)$
$T_7 + T_8 + T_9 \le 2T_1 \Rightarrow 1 + 2 + 3 \le 2(4)$
$T_3 + T_4 - T_5 \le 2T_1 \Rightarrow 4 + 5 - 3 \le 2(4)$
$T_5 + T_6 - T_3 \le 2T_1 \Rightarrow 3 + 6 - 4 \le 2(4)$
$T_2 \le T_1 \Rightarrow 3 \le 4$, $T_3 \le 2T_1 \Rightarrow 4 \le 2(4)$, $T_5 \le 2T_1 \Rightarrow 3 \le 2(4)$, $T_6 \le 2T_1 \Rightarrow 6 \le 2(4)$,
$T_7 \le 2T_1 \Rightarrow 1 \le 2(4)$, $T_8 \le 2T_1 \Rightarrow 2 \le 2(4)$, $T_{10} \le 2T_1 \Rightarrow 4 \le 2(4)$

Thus all the time constraints can be met.

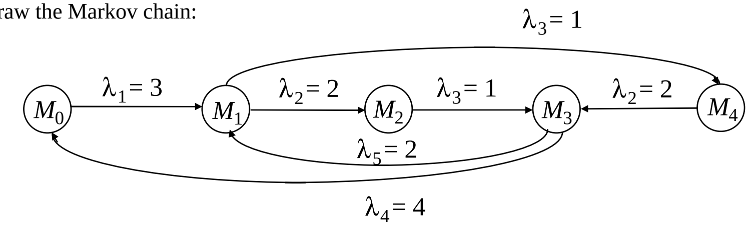
2. Find the reachability set:

$M_0 = (1, 0, 0, 0, 0, 0)$ $M_1 = (0, 1, 1, 0, 0, 1)$ $M_2 = (0, 0, 1, 1, 0, 1)$
$M_3 = (0, 0, 0, 1, 1, 1)$ $M_4 = (0, 1, 0, 0, 1, 1)$

And the reachability tree:



Draw the Markov chain:



Applying a 'flow' balance:

$3P[M_0] = 4P[M_3]$
$3P[M_1] = 2P[M_3] + 3P[M_0]$
$P[M_2] = 2P[M_1]$
$6P[M_3] = P[M_2] + 2P[M_4]$
$2P[M_4] = P[M_1]$
$P[M_0] + P[M_1] + P[M_2] + P[M_3] + P[M_4] = 1$

Solve these equations to give:

$P[M_0] = 0.1429$ $P[M_1] = 0.2143$ $P[M_2] = 0.4286$
$P[M_3] = 0.1071$ $P[M_4] = 0.1071$

and the token occupancy probabilities:

$P[\mu_1=1] = P[M_0] = 0.1429$ $P[\mu_2=1] = P[M_1] + P[M_4] = 0.3214$
$P[\mu_3=1] = P[M_1] + P[M_2] = 0.6429$ $P[\mu_4=1] = P[M_2] + P[M_3] = 0.5357$
$P[\mu_5=1] = P[M_3] + P[M_4] = 0.2412$
$P[\mu_6=1] = P[M_1] + P[M_2] + P[M_3] + P[M_4]= 0.8571$

As we have token conservation, Little's law can be applied. The utilization of t_1 is $P[\mu_1=1] = 0.1429$ and as $\lambda_1 = 3$ the average token flow in p_1 is 0.4287 tokens/unit time. Due to the fork at t_1, the flow through the subsystem (composed of p_2, p_3, p_4, p_5, p_6 and t_2, t_3, t_4, t_5) is:

$\bar{\lambda} = 1.2861.$

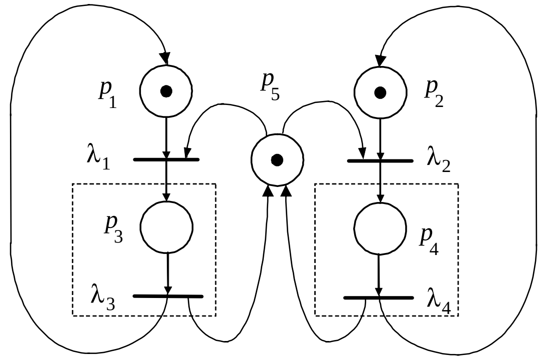
The average number of tokens in the subsystem is given by :

$\bar{N} = \bar{\mu}_2 + \bar{\mu}_3 + \bar{\mu}_4 + \bar{\mu}_5 + \bar{\mu}_6 = 2.5983$

The average time a token takes to return to p_1 is given by:

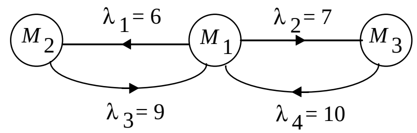
$$\bar{T} = \frac{\bar{N}}{\bar{\lambda}} = 2.02 \text{ time units}$$

3.



The reachability set is:
$M_1 = (1, 1, 0, 0, 1)$
$M_2 = (0, 1, 1, 0, 0)$
$M_3 = (1, 0, 0, 1, 0)$

and the corresponding Markov chain:



Applying a flow balance to the marking probabilities gives:

$13P[M_1] = 9P[M_2] + 10P[M_3]$
$9P[M_2] = 6P[M_1]$
$10P[M_3] = 7P[M_1]$
$P[M_1] + P[M_2] + P[M_3] = 1$

Solve to give:

$P[M_1] = 0.4225$ $P[M_2] = 0.2817$ $P[M_3] = 0.2958$

and the token occupancy probabilities are:

$P[\mu_1 = 1] = P[M_1] + P[M_3] = 0.7183$
$P[\mu_2 = 1] = P[M_1] + P[M_2] = 0.7042$
$P[\mu_3 = 1] = P[M_2] = 0.2817$
$P[\mu_4 = 1] = P[M_3] = 0.2958$
$P[\mu_5 = 1] = P[M_1] = 0.4225$

As t_1 requires tokens at p_1 and p_5 to be enabled, the utilization is:

$P[\mu_1 = 1]P[\mu_5 = 1] = 0.3035$

As $\lambda_1 = 6$, the average token flow into p_3 (and hence into subsystem 1) is:

$\bar{\lambda}_1 = 6 \times 0.3035 = 1.8209$

The average number of tokens in subsystem 1 is $\bar{N}_1 = P[\mu_3 = 1] = 0.2817$

Thus the average time a token spends in p_3 (and hence the average time that the 1st machine assembles a part in process 1) is given by:

$$\bar{T}_1 = \frac{\bar{N}_1}{\bar{\lambda}_1} = 0.1547 \text{ time units}$$

The same approach is used for process 2 to give: $\bar{T}_2 = \frac{\bar{N}_2}{\bar{\lambda}_2} = 0.1420$ time units

Thus process 1 is active 0.1547/(0.1547+0.1420) = 52.1% of the assembly time.